

**MATRIUS, DETERMINANTS I SISTEMES**

1. Estudiau i resoleu els següents sistemes :

$$\left. \begin{array}{l} x+3y-z=-11 \\ x-y-2z=0 \\ x+y+z=2 \end{array} \right\} \begin{array}{l} \text{b)} \left. \begin{array}{l} x+3y-z+t=0 \\ 2x+5y-t=0 \\ 3x+4z+t=0 \\ 2x-2y+z-2t=0 \end{array} \right\} \end{array} \left. \begin{array}{l} 3x+2y-z=0 \\ 4x+5y-3z=0 \\ 2x-y+z=0 \\ x+3y-2z=0 \end{array} \right\} \text{c)}$$

2. Donat el sistema d'equacions :

$$\left. \begin{array}{l} x \cos a + y \sin a = 1 \\ x \sin a - y \cos a = 1 \end{array} \right\}$$

1. Resoleu, obtenint x e y en funció de "a".

2. Calculeu "a" per a que x+y=1.

3. Trobau "m" per a que el sistema tenguí infinites solucions :

$$\left. \begin{array}{l} x+2y+3z=0 \\ 3x+2y-z=0 \\ 5x+6y+mz=0 \end{array} \right\}$$

4. Estudiau els següents sistemes segons el valor del paràmetre :

$$\left. \begin{array}{l} (m+1)x+y+z=m^2+3m \\ a) x+(m+1)y+z=m^3+3m^2 \\ x+y+(m+1)z=m^4+3m^3 \end{array} \right\} \begin{array}{l} 5x-11y+9z=m \\ b) x-3y+5z=2 \\ 2x-4y+2z=1 \end{array}$$

5. Estudiau els següents sistemes, segons els valors del paràmetre i resoleu quan sigui compatible :

$$\left. \begin{array}{l} 2x+3y-4z=1 \\ a) 4x+6y-mz=2 \\ x+y+mz=10 \end{array} \right\} \begin{array}{l} 2y-z=k \\ b) \left. \begin{array}{l} 3x-2z=11 \\ y+z=6 \\ 2x+y-4z=k \end{array} \right\} \end{array} \left. \begin{array}{l} x+y+z=a \\ c) x+(1+a)y+z=2a \\ x+y+(1+a)z=0 \end{array} \right\} \begin{array}{l} ax+y+z=a \\ d) x+ay+z=a \\ x+y+az=a \end{array}$$

6. Estudiau el sistema segons els valors d'm:

$$\left. \begin{array}{l} x+y+3z=-1 \\ 2x+5y+4z=-2 \\ x+3y+m^2z=m \end{array} \right\} \text{Resoleu per } m=-1$$

7. Estudiau els següents sistemes segons els distints valors de "a" i "b" :

$$\left. \begin{array}{l} 2x+y=a \\ a) 4x+2y=1+b \\ 5x+3y=2 \end{array} \right\} \left. \begin{array}{l} 3x-y+2z=1 \\ b) x+4y+z=b \\ 2x-5y+az=-2 \end{array} \right\} \left. \begin{array}{l} x+ay+a^2z=1 \\ c) x+ay+abz=a \\ bx+ay+a^2bz=a^2b \end{array} \right\}$$

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**SOLUCIONS :**

1.a)  $\text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det. ;  $x=1, y=-3, z=4$

**b)**  $\text{rg}(A)=4=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Hom. Incomp. (només admet  $x=y=z=t=0$ )

**c)**  $\text{rg}(A)=2=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Hom. Comp. ;  $x=-z/7, y=5z/7$

2. a)  $\text{rg}(A)=2=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det.  $X=\cos a + \sin a$   
 $y=-\cos a + \sin a$

b)  $a = \frac{\pi}{6} + 2k\pi \quad k \in \mathbb{Z}$

3.  $m=5$

4. a)  $m \neq 0, -3 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det.

$m=0 \Rightarrow \text{rg}(A)=1=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Homog. Comp.

$m=-3 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Homog. Comp.

**b)**  $m \neq 4 \Rightarrow \text{rg}(A)=2 \neq 3=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

$m=4 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Comp. det.

5.a)  $m=8 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Comp ind. ;  $x=29-28z,$   
 $y=-19+20z, z=z$

$m \neq 8 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det. ;  $x=29, y=-19, z=0$

**b)**  $k=6 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det. ;  $x=5, y=4, z=2$

$k \neq 6 \Rightarrow \text{rg}(A)=3 \neq 4=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

**c)**  $a=0 \Rightarrow \text{rg}(A)=1=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. Homo. Comp. ind. ;  $x=-y-z, y=y$   
 $z=z$

$a \neq 0 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. comp. det.  $x=a, y=1, z=-1$

**d)**  $a \neq 1, -2 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det. ;  $x=y=z=a/(a+2)$

$a=1 \Rightarrow \text{rg}(A)=1=\text{rg}(A^*)<n$ . incóg.  $\Rightarrow$  sis. comp. ind. ;  $x=1-y-z, y=y, z=z$

$a=-2 \Rightarrow \text{rg}(A)=2 \neq 3=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

6. per  $m = \pm \sqrt{\frac{5}{3}}$ , sistema incompatible. Compatible determinat en els altres casos. Per  $m=-1, x=-1, y=0, z=$

0.

7. a)  $2a-b-1 \neq 0 \Rightarrow \text{rg}(A)=2 \neq 3=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

$2a-b-1=0 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det.

**b)**  $\forall b \ a \neq 0, 1 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. indet.

$a=0 \ b \neq 2 \Rightarrow \text{rg}(A)=2 \neq 3=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. Incomp.

$a=0 \ b=2 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. comp. det.

$a=1 \ b \neq 0, 1 \Rightarrow \text{rg}(A)=2 \neq 3=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

$a=1 \ b=0, 1 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. Comp. indet.

**c)**  $a \neq 0, b \ i \ b \neq 1 \Rightarrow \text{rg}(A)=3=\text{rg}(A^*)=n$ . incóg.  $\Rightarrow$  sis. Comp. det.

$a=0 \Rightarrow \text{rg}(A)=1 \neq 2=\text{rg}(A^*) \Rightarrow$  sis. Incomp.

$a=b=1 \Rightarrow \text{rg}(A)=1=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. Comp. det.

$a=-1 \ b=1 \Rightarrow \text{rg}(A)=2=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. Comp. indet.

$a=b \ a \neq 0, 1 \Rightarrow \text{rg}(A)=\text{rg}(A^*) < n$ . incóg.  $\Rightarrow$  sis. Comp. indet.